| $y = a + b \cdot x \Rightarrow b = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$ | | | | | $\frac{0}{0} \frac{\infty}{\infty} 0 \cdot \infty \infty - \infty 0^0 1^\infty \infty^0$ | | | |
|--|---|--|---|---|--|--|--|--|
| $0 \cdot const = 0$ | $\frac{const}{\infty} = 0$ | $\frac{const}{\infty} = 0 const^{\infty} = \infty const \pm \infty = \pm \infty$ | | | | | | |
| | con | $st + \infty =$ | $=\infty$ cons | $t - \infty = -\infty$ | $const^{\infty}$ | = ∞ | | |
| r > 0 | | 0 | 1 | 2 | h a | +h | | χ^a and a b |
| $y = a^x, a > 0$ | | $x^0 = $ | 1 | χ^a . | $x^{\mathrm{b}} = x^{\mathrm{a}}$ | | | $\frac{1}{x^{b}} = x^{a-b}$ |
| $(x^{a})^{b} = x^{a \cdot b}$ | | | $x^{\frac{a}{b}} = ($ | $\sqrt[b]{x}^{a} = \sqrt[b]{x^{a}}$ | $x^{-a} = \frac{1}{x^{a}}$ | | | $a^{a} = \frac{1}{x^{a}}$ |
| $y = \log_a(x), a > 0 \text{ i } a \neq 1$ | x > 0 | | $\log_a(x) =$ | $y \Leftrightarrow x = a^y$ | $x \Leftrightarrow x = a^y \qquad \log_a(x)$ | | $(x \cdot y) = l$ | $og_a(x) + log_a(y)$ |
| $\log_{a}\left(\frac{x}{y}\right) = \log_{a}(x) - \log_{a}(y)$ | $g_a(y)$ $\log_a[(x)^n] = n \cdot \log_a(x)$ | |) $\log_a(x)$ | $\log_{a}(x) = \frac{\log_{b}(x)}{\log_{b}(a)}$ | | log log | $g_e(x) = \ln(x)$ $_{10}(x) = \log(x)$ | |
| | n n | | | | | | | |
| $a_1 + a_2 + a_3 + \dots + a_n =$ | $\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} a_i$ | a_i | n! = 2 | $1 \cdot 2 \cdot 3 \cdot \dots (n -$ | - 1) · n | | $\binom{n}{k} =$ | $\frac{n!}{k! \cdot (n-k)!}$ |
| $(x+y)^n = \binom{n}{0} \cdot x^n \cdot y^0$ | $(n) + \binom{n}{1} \cdot x$ | $n-1 \cdot y^1$ | $+\binom{n}{2} \cdot x^{n}$ | $x^{n-2} \cdot y^2 + \dots +$ | $\binom{n}{n-1}$ | $\cdot x^{n-(n-1)}$ | $(y^{n-1} + y^{n-1}) + (y^{n-1}) + (y^{n-$ | $\binom{n}{n} \cdot x^{n-n} \cdot y^n$ |
| | | | 1.0 | | | | | |
| $p.w.s[a;b] = \frac{f(b) - f(a)}{b - a} = \frac{\Delta f}{\Delta x}$ | | | $f'(a) = \frac{df}{dx} = \lim_{b \to a} \left(\frac{f(b) - f(a)}{b - a} \right)$ | | | $f'(a) = \lim_{\Delta x \to 0} \left(\frac{f(a + \Delta x) - f(a)}{\Delta x} \right)$ | | |
| $f(x) = \mathbf{a} \cdot g(x) \qquad \qquad f(x)$ | | | $f(x) = g(x) \pm h(x) \qquad \qquad h(x) = f(x) \cdot g(x) \\ \Downarrow \qquad \qquad \Downarrow$ | | | | | |
| $f'(x) = \mathbf{a} \cdot g'(x) \qquad \qquad f'(x) = g$ | | | $g'(x) \pm h'(x)$ | $h'(x) \pm h'(x)$ $h'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$ | | | | |
| $h(x) = \frac{f(x)}{g(x)} \Rightarrow h'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2}$ | | | | f(| $f(x) = z(w(x)) \Rightarrow f'(x) = z'(w) \cdot w'(x)$ | | | |
| $(x^{n})' = n \cdot x^{n-1}$ $(a^{x})' = a$ | $a^x \cdot \ln(a)$ $(e^x)' = e^x$ | | | $[\log_a(x)]'$ | $[\log_a(x)]' = \frac{1}{x} \cdot \log_a(e) = \frac{1}{x \cdot \ln(a)} \qquad [\ln(x)]$ | | | $[\ln(x)]' = \frac{1}{x}$ |
| (const)' = 0 $[cos(x)]'$ | | | $=-\sin(x)$ $[\sin(x)]' = \cos(x)$ | | | | | |
| $[\operatorname{tg}(x)]' = \frac{1}{\cos^2(x)}$ | | | | | $[\operatorname{ctg}(x)]' = -\frac{1}{\sin^2(x)}$ | | | |
| $\operatorname{grad} f(x) = \frac{\mathrm{d}f}{\mathrm{d}x} \cdot \hat{x}$ $\operatorname{grad} f(x, y, z) = \frac{\partial f}{\partial x} \cdot \hat{x} + \frac{\partial f}{\partial y} \cdot$ | | | $\hat{y} + \frac{\partial f}{\partial z} \cdot \hat{z}$ $\frac{\partial f(x, y)}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x}$ | | | | | |
| | | | | | | | | |
| $\Delta F = \pm \sum_{i=1}^{m} \left \frac{\partial f}{\partial A_i} \right \cdot \Delta A_i \qquad A_i$ | $\Delta f = f' +$ | $\Delta x = \Big $ | $\left \frac{\mathrm{d}f}{\mathrm{d}x}\right \cdot \Delta x$ | $y = \mathbf{a} \cdot x + \mathbf{b}$ | a = | $\lim_{x\to\infty} \left[\frac{f(x)}{x}\right]$ | <u>()</u>] b | $=\lim_{x\to\infty}[f(x)-\mathbf{a}\cdot x]$ |
| $\mathrm{d}y = y' \cdot \mathrm{d}x$ | $\mathrm{d}y = y' \cdot \mathrm{d}x \qquad \qquad [f'(x)]'$ | | |]' = f''(x) | | | $\left[f^{(n)}(x)\right]'$ | $=f^{(n+1)}(x)$ |
| $f(x) = f(a) + \sum_{i=1}^{\infty} \frac{f^{(i)}(a)}{i!} \cdot (x-a)^{i}$ | | | | $f(x) = f(0) + \sum_{i=1}^{\infty} \frac{f^{(i)}(0)}{i!} \cdot x^{i}$ | | | | |
| $v(t) = \frac{\mathrm{d}x}{\mathrm{d}t}$ | $a(t) = \frac{d}{dt}$ | $a(t) = \frac{\mathrm{d}v}{\mathrm{d}t} \qquad F(t)$ | | | $F(x) = \frac{\mathrm{d}W}{\mathrm{d}x} \qquad E$ | | $E(x) = -\frac{\mathrm{d}V}{\mathrm{d}x}$ | |
| $\text{Jeżeli}\lim_{x \to x_0} f(x) = 0 \text{ oraz } \lim_{x \to x_0} g(x) = 0 \text{ lub } \lim_{x \to x_0} f(x) = \infty \text{ oraz } \lim_{x \to x_0} g(x) = \infty, \text{ to } \lim_{x \to x_0} \left[\frac{f(x)}{g(x)} \right] = \lim_{x \to x_0} \left[\frac{f'(x)}{g'(x)} \right]$ | | | | | | | | |

Strona **2**

| $F(x) = \int f(x) \mathrm{d}x \Leftrightarrow F'(x)$ | = f(x) | $\int c \cdot f(x) \mathrm{d}x = c \cdot$ | $\int f(x)\mathrm{d}x$ | $\int [f(x) + g(x)]$ | $\mathrm{d}x = \int f(x)\mathrm{d}x + \int g(x)\mathrm{d}x$ | |
|---|---|--|------------------------|--|---|--|
| $\int f(x) \cdot g'(x) \cdot \mathrm{d}x =$ | $f(x) \cdot g(x)$ | $\int u \cdot \mathrm{d}v = u \cdot v - \int v \cdot \mathrm{d}u$ | | | | |
| $\int x^{n} dx = \frac{1}{n+1} \cdot x^{n+1} + C$ | $\int x^{n} dx = \frac{1}{n+1} \cdot x^{n+1} + C$ $\int x^{-1} dx = \ln x + C$ | | | $\int a^{x} dx = \frac{a^{x}}{\ln(a)} + C \qquad \qquad \int e^{x} dx = e^{x} + C$ | | |
| $\int \ln(x) \mathrm{d}x = x$ | x + C | $\int \sin(x) \mathrm{d}x = -\cos(x) + C$ | | | | |
| $\int \cos(x) \mathrm{d}x$ | + C | $\int \frac{1}{\cos^2(x)} \mathrm{d}x = \mathrm{tg}(x) + \mathrm{C}$ | | | | |
| $\int_{a}^{b} f(x) \mathrm{d}x = F(x)$ | - <i>F</i> (a) | $\int_{a}^{b} f(x) dx = -\int_{b}^{a} f(x) dx$ | | | | |

| $i^{2} = -1 \Rightarrow \sqrt{-1} = i$ Dla $z_{1} = a_{1} + i \cdot b_{1}$ $z_{2} = a_{2} + i \cdot b_{2}$ $z_{1} + z_{2} = a_{1} + a_{2} + i \cdot (b_{1} + b_{2})$ $z_{1} + z_{2} = a_{1} + a_{2} + i \cdot (b_{1} + b_{2})$ | $i^2 = -1 \Rightarrow \sqrt{-1} = i$ | Dla $z_1 = a_1 + i \cdot b_1$ $z_2 = a_2 + i \cdot b_2$ | $z_1 = a_1 + i \cdot b_1$ | $z_1 = z_2 \Leftrightarrow a_1 = a_2 \land b_1 = b_2$ | | |
|---|--------------------------------------|--|---|---|--|--|
| $z_1 \cdot z_2 = a_1 \cdot a_2 - b_1 \cdot b_2 + i \cdot (a_1 \cdot b_2 + b_1 \cdot a_2)$ | | | | $z_1 + z_2 = a_1 + a_2 + i \cdot (b_1 + b_2)$ | | |
| | | | $z_1 \cdot z_2 = a_1 \cdot a_2 - b_1 \cdot b_2 + i \cdot (a_1 \cdot b_2 + b_1 \cdot a_2)$ | | | |

| | $z = a + i \cdot b = [r, \varphi]$ | $= \mathbf{r} \cdot [\cos(\varphi) + i \cdot \sin(\varphi)] = \mathbf{r} \cdot e^{i \cdot \varphi}, \text{ gdzie } z = \mathbf{r} = \sqrt{a^2 + b^2}, \ \cos(\varphi) = \frac{a}{r}, \sin(\varphi) = \frac{b}{r}$ |
|---|--|---|
| Dla $z_1 = [r_1, \varphi_1] - z_2 = [r_2, \varphi_2]$ | $z_1 \cdot z_2 = [r_1 \cdot r_2, \varphi_1 + \varphi_2] = r_1 \cdot r_2 \cdot [\cos(\varphi_1 + \varphi_2) + i \cdot \sin(\varphi_1 + \varphi_2)]$ | |
| | $z_1 = [r_1, \phi_1]$ $z_2 = [r_2, \phi_2]$ | $\frac{z_1}{z_2} = \left[\frac{r_1}{r_2}, \phi_1 - \phi_2\right] = \frac{r_1}{r_2} \cdot \left[\cos(\phi_1 - \phi_2) + i \cdot \sin(\phi_1 - \phi_2)\right]$ |
| | | $z = [r, \phi] \Rightarrow z^{n} = [r^{n}, n \cdot \phi] = r^{n} \cdot [\cos(n \cdot \phi) + i \cdot \sin(n \cdot \phi)]$ |

| $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} =$ | = [a _{ij}] _{m×n} | $A^T =$ | $\left[\mathbf{a}_{ji}\right]_{\mathbf{n}\times\mathbf{m}} \qquad \mathbf{k}\cdot A = \left[\mathbf{k}\cdot\mathbf{a}_{ij}\right]_{\mathbf{m}\times\mathbf{n}}$ | | | |
|--|---|-----------------------------------|---|--|--|--|
| $A = \left[a_{ij}\right]_{m \times n} B = \left[b_{ij}\right]_{m \times n} \Rightarrow A + B = \left[a_{ij} + b_{ij}\right]_{m \times n}$ | | | $A = \left[a_{ij}\right]_{m \times n} B$ | $B = \left[\mathbf{b}_{ij}\right]_{n \times p} \Rightarrow A \cdot B = \left[\sum_{l=1}^{n} \mathbf{a}_{il} \cdot \mathbf{b}_{lj}\right]_{m \times p}$ | | |
| Wyznacznik | | | Dopełnienie | | | |
| $W = \det(A) = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ \vdots & \vdots \\ \vdots & \vdots \\ a_{n1} & a_{n2} \end{vmatrix}$ | $\det(A) = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} \qquad A_{jk} = (-1)^{jk}$ | | | $(-1)^{j+k} \cdot M_{jk} = (-1)^{j+k} \cdot \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1k} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2k} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ a_{j1} & a_{j2} & \cdots & a_{jk} & \cdots & a_{jn} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nk} & \cdots & a_{nn} \end{vmatrix}$ | | |
| $W = \det(A) = \sum_{m}$ | $\sum_{n=1}^{n} a_{wm} \cdot A_{wn}$ | m | $W = \det(A) = \sum_{m=1}^{n} a_{mk} \cdot A_{mk}$ | | | |
| Macierz odwrotna, A^{-1} | | $A^{-1} \cdot A = A \cdot A^{-1}$ | I = I | $A^{-1} = \frac{1}{\det(A)} \cdot \left[A_{jk}\right]^{T}$ | | |
| Równanie macierzowe | | $A \cdot X = B$ | | $X = A^{-1} \cdot B$ | | |